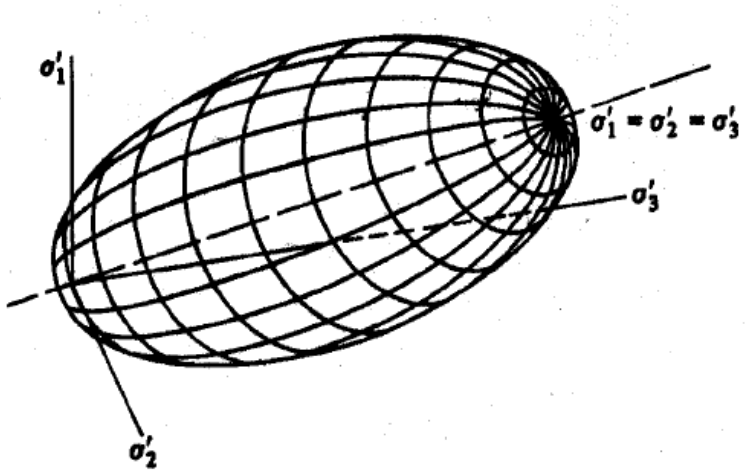


Geomechanics

LECTURE 8

H-M COUPLED FORMULATION

DR. ALESSIO FERRARI

Laboratory of soil mechanics – Fall 2024

11.11.2024

Access the QUIZ

https://etc.ch/atKk

Geomechanics
General Course overview

Topics

Basic concepts

Elasticity - Linear & Non-linear

Plasticity Perfect-plasticity

Hardening elasto-plasticity

Critical state concept

Classical constitutive framework

Modified Cam-Clay

Practical aspects

Geomechanics in Practice

Numerical modelling

In-situ stress

Course Project

Retaining structures

Advanced constitutive models

Unsaturated behavior

THM behaviour of geomaterials

Bio-cementation

Time-dependent behaviour

Content

- H-M coupled formulation
  - Sign convention
  - Formulation for water/solid skeleton interactions
  - Examples
- Conclusion

H-M coupled formulation

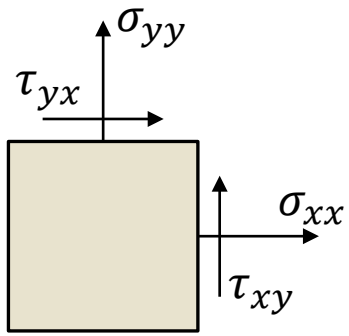
SIGN CONVENTION

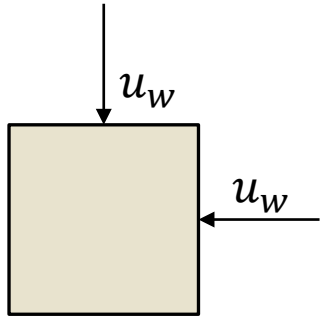
FORMULATION FOR WATER/SOLID SKELETON INTERACTIONS (H-M PROBLEMS)

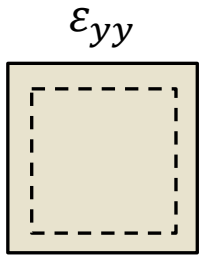
EXAMPLES

Sign conventions

Sign conventions to be checked for each formulation / numerical code:







Positive for extension

Variables (n. of unknowns)

σ_{ij} : total stresses (6)

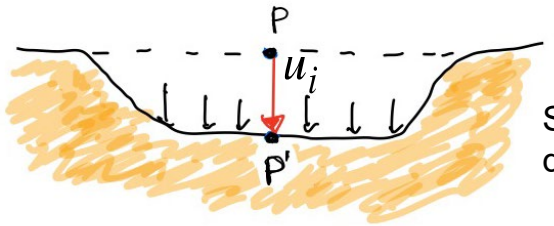
u_w : pore water pressure (1)

σ'_{ij} : effective stresses (6)

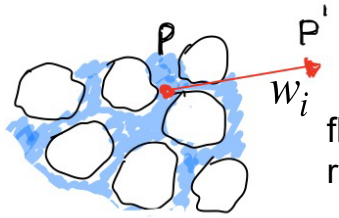
ε_{ij} : deformations (6)

u_i : solid skeleton displacements (3)

w_i : fluid displacement (3)



Solid displacement



fluid displacement relative to the solid phase

25 unknowns vs 25 equations

Equations (n. of equations)

Momentum (3)

$$\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0$$ (no acceleration terms), f_i are body forces

Effective stress definition (6)

$$\sigma'_{ij} = \sigma_{ij} + u_w \delta_{ij}$$ (Terzaghi's definition, saturated soil)

Constitutive relationship (6)

$$\dot{\sigma}'_{ij} = D_{ijhk} \dot{\epsilon}_{hk}$$

Compatibility of displacements (6)

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Darcy's law (3)

$$\dot{w}_i = -k_{ij}\frac{\partial}{\partial x_j}\left(\frac{u_w}{\gamma_w} + z_e\right)$$

Continuity of fluid phase (1)

$$\frac{\partial \dot{w}_i}{\partial x_i} = -\dot{\epsilon}_{vol}$$ (compressibility of fluid and solid neglected)

Example 1 : Dry soil

σ_{ij} : total (= effective) stresses (6)

ϵ_{ij} : deformations (6)

u_i : solid skeleton displacements (3)

Momentum (3)

$$\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0$$ (no acceleration terms), f_i are body forces

Constitutive relationship (6)

$$\delta \sigma'_{ij} = D_{ijhk} \delta \epsilon_{hk}$$

Compatibility of displacements (6)

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

15 unknowns vs 15 equations

Example 2 : Saturated soil subjected to change in total stresses

The new total stress field must be equilibrated:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0$$

Changes in total stresses imply (general case) changes in effective stresses and pore water pressure:

$$\sigma'_{ij} = \sigma_{ij} + u_w \delta_{ij}$$

Changes in pore water pressure imply change in fluid velocity:

$$\dot{w}_i = - k_{ij} \frac{\partial}{\partial x_j} \left(\frac{u_w}{\gamma_w} + z_e \right)$$

Deformations are linked to changes in effective stresses

$$\dot{\sigma}'_{ij} = D_{ijhk} \dot{\epsilon}_{hk}$$

Continuity eq. links change in fluid velocities to deformations:

$$\frac{\partial \dot{w}_i}{\partial x_i} = - \dot{\epsilon}_{vol}$$

Need to solve all the equations simultaneously: Hydro-Mechanical coupled problem (e.g. Consolidation)

Thank you for your attention